The thermal energy of a scalar field on a unidimensional Riemann surface

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Abstract

We discuss some controverted aspects of the evaluation of the thermal energy of a scalar field on a unidimensional Riemann surface. The calculations are carried out using a generalised zeta function approach.

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1 Introduction

In a recent paper Brevik, Milton and Odintsov [1] calculated the thermal energy associated with several types of quantum fields living in $S^1 \times S^{d-1}$ geometries with $d = 1, 2$. In particular the thermal energy of a quantum neutral scalar field defined on a unidimensional Riemann surface at finite temperature ($S^1 \times S^1$) was found by those authors to depend linearly on the temperature in the high-temperature limit. Their result reads

$$E(\beta) \simeq \frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^2 - \frac{1}{2\beta}, \qquad a/\beta \gg 1, \tag{1}$$

where β is the reciprocal of the thermal equilibrium temperature T. This expression is in agreement with the corresponding one obtained by Kutasov and Larsen [2] (in the high-temperature limit) but at variance with the one obtained by Klemm *et al* [3]. These last authors justify their result by arguing that the contribution of the zero modes becomes significant only when the saddle-point approximation does not hold anymore. This disagreement has arisen some doubts concerning the correctness of Brevik *et al*'s evaluation. It was argued by Dowker that the correct result for the thermal energy for this particular arrangement of quantum field and geometry contains a positive, linearly-dependent term in the temperature that in the high-temperature limit cancels exactly the corresponding negative term in (1), see [4] and references therein. This same result can be inferred from Cardy's analysis of partition functions for conformal field theories in higher dimensions [5]. Besides a fundamental problem concerning the associated entropy function, this disagreement may have consequences for the Verlinde-Cardy relations [6, 7].

The purpose of this brief note is to examine such questions a little bit further. We will see that a generalised zeta function approach to the problem shows that: (i) there is indeed a contribution to the thermal energy of the spatial zero mode as calculated independently by Dowker [8] and Dowker and Kirsten [9], and also Cardy [5]; (ii) though the thermal energy is independent of the scaling mass inherent to zeta function regularization, the free energy and the entropy associated with the spatial zero mode depend on the scaling mass; and finally, contrary to what is argued in [8], (iii) that we cannot discard the terms that depend on the scaling mass without contradicting fundamental thermodynamical relationships or, to put it in other words, that we cannot choose the value of the scaling mass in such a way that the third law of thermodynamics is obeyed by the thermodynamics of the zero mode.

2 Zeta function approach to the partition function

We begin by briefly sketching a generalised zeta function [10] approach to the partition function of a quantum scalar field in thermal equilibrium with a heat bath at a fixed temperature β^{-1}.

The zeta function for conformally coupled massless scalars fields living on a d_R-dimensional Riemann surface at finite temperature reads

$$
\begin{aligned}
\zeta\left(s;\frac{\partial^2+\xi R}{\mu^2}\right) &= \mathrm{Tr}\left(\frac{\partial^2+\xi R}{\mu^2}\right)^{-s} \\
&= \mu^{2s}\sum_{\lambda}\left(\omega_\lambda^2\right)^{-s}, \qquad (2)
\end{aligned}
$$

where $\partial^2 := \nabla^2 + \partial^2/\partial^2\tau$, μ is a scaling mass, $\xi = (D-2)/4(D-1)$ is the conformal parameter and R is the Ricci curvature scalar. Here $D = d_R + 1$ where d_R denotes the number of dimensions of the Riemann surface. The eigenvalues ω_λ^2 of the Euclidean d'Alembertian operator are given by

$$
\omega_\lambda^2 = \left(\frac{2\pi n}{\beta}\right)^2 + \left(\frac{M_\ell^2}{a^2}\right)^2, \qquad (3)
$$

where, in principle, $n, \ell \in Z = \{0, \pm 1, \pm 2, \ldots\}$. Here M_ℓ^2/a^2 denote the eigenvalues of the Laplace operator on the d_R-dimensional Riemann surface. For a bosonic neutral scalar field the partition function can be obtained from the zeta function through

$$
\log Z(\beta) = \frac{1}{2}\zeta'\left(s=0;\frac{\partial^2+\xi R}{\mu^2}\right). \qquad (4)
$$

We can separate the zero temperature sector which corresponds to $n = 0$, i.e., the zero mode associated with the compactified Euclidean temporal dimension, from the finite temperature sector and at the same time isolate the spatial zero mode in the following way. Suppose that the spatial zero mode corresponds to $\ell = 0$ and the remaining ℓ are shifted to $\ell + 1$ such that $\ell \in N = \{0, 1, 2, \ldots\}$ with an appropriate degeneracy factor taken into account. For convenience, however, suppose initially that the spatial zero mode is absent. Then taking (3) into (2), performing a Mellin transform, and making use of the Poisson sum formula

$$
\sum_{n\in Z} e^{-n^2\pi\tau} = \frac{1}{\sqrt{\tau}}\sum_{n\in Z} e^{-n^2\pi/\tau}, \qquad (5)
$$

with $\tau = 4\pi t/\beta^2$, we have

$$
\begin{aligned}
\zeta\left(s;\frac{\partial^2+\xi R}{\mu^2}\right) &= \frac{\mu^{2s}}{\Gamma(s)}\int_0^\infty dt\, t^{s-1}\sum_{\ell\in N} D_\ell e^{-\left(M_\ell^2/a^2\right)t}\sum_{n\in Z} e^{-\left(4\pi^2 n^2/\beta^2\right)t} \\
&= \frac{\mu^{2s}}{\Gamma(s)}\frac{\beta}{\sqrt{4\pi}}\sum_{\ell\in N} D_\ell\int_0^\infty dt\, t^{s-3/2} e^{-\left(M_\ell^2/a^2\right)t} + \frac{\mu^{2s}}{\Gamma(s)}\frac{\beta}{\sqrt{\pi}}\sum_{\ell\in N} D_\ell \\
&\times \sum_{n\in N_0}\int_0^\infty dt\, t^{s-3/2} e^{-n^2\beta^2/4t - M_\ell^2/a^2 t}, \qquad (6)
\end{aligned}
$$

where $\mathrm{N}_0 := N - \{0\}$ and D_ℓ is the degeneracy factor. It is not hard to see that the spatial zero mode can be taken into account by considering the additional term

$$\zeta_{Z.M.}\left(s;\frac{\partial^2+\xi R}{\mu^2}\right)=\frac{\mu^{2s}}{\Gamma(s)}\frac{\beta}{\sqrt{\pi}}\sum_{n\in N_0}\int_0^\infty dt\, t^{s-3/2}e^{-n^2\beta^2/4t}. \tag{7}$$

The contribution of the spatial zero mode as it stands is divergent, the origin of this divergence being the simultaneous consideration of two zero modes, the temporal and the spatial ones. Nevertheless, we can regularise it and extract its finite contribution in several ways. One of this ways is to apply the Poisson sum formula under the form

$$\sum_{n\in N_0} e^{-n^2\pi/\tau}=-\frac{1}{2}+\frac{\sqrt{\tau}}{2}+\sqrt{\tau}\sum_{n\in N_0} e^{-n^2\pi\tau}. \tag{8}$$

Then, discarding terms that do not depend on β or contribute to the partition function with a term which is linear in β, we obtain

$$\zeta_{Z.M.}\left(s;\frac{\partial^2+\xi R}{\mu^2}\right)=2\left(\frac{\mu\beta}{2\pi}\right)^{2s}\zeta_R(2s). \tag{9}$$

Since $\zeta_R(0)\neq 0$, we can expect a scaling mass dependence of some or all thermodynamical quantities associated with the spatial zero mode. The derivative at $s=0$ reads

$$\zeta'_{Z.M.}\left(s=0;\frac{\partial^2+\xi R}{\mu^2}\right)=4\log\left(\frac{\mu\beta}{2\pi}\right)\zeta_R(0)+4\zeta'(0). \tag{10}$$

The last term is linear in β and can be discarded, as we did above.

An alternative way of treating the thermal contribution of the zero mode is to proceed along the lines pioneered in [11], that is (in a nutshell) to use the ordinary formulas of zeta regularization till the end, while 'lifting' the zero modes (e.g. preventing them from becoming zero), both the spatial and temporal one in this case, with the help of two small parameters (say ε and η), which are finally taken to zero after the whole zeta regularization process (which uses the ordinary expressions in the absence of zero modes) has been carried through. This method provides, in some situations, a justification of the usual principal part prescription (see [11]), and in the present case leads also to an elegant cancellation of precisely the same divergences that we have just discarded on physical (dimensional) grounds.

Making use of the integral representation for the modified Bessel function of the third kind [12]

$$\int_0^\infty dx\, x^{\nu-1}e^{-\frac{a}{x}-bx}=2\left(\frac{a}{b}\right)^{\nu/2}K_\nu\left(2\sqrt{ab}\right), \tag{11}$$

for the temperature-dependent part describing the non-zero modes, and adding the zero-mode contribution, we have

$$\log\, Z\left(\beta\right)=-\frac{\beta}{2a}\sum_{\ell\in N-\{1\}}D_\ell M_\ell-\log\left(\frac{\mu\beta}{2\pi}\right)-\sum_{\ell\in N-\{1\}}D_\ell\log\left(1-e^{-\beta M_\ell/a}\right), \tag{12}$$

where we have also used the explicit form of $K_{1/2}\left(z\right)$ [12]. Notice that in this approach —and also in the one described in [11]— there is no temperature dependent pole as in [8].

Equation (12) can be applied to several situations concerning scalar conformally coupled fields on a d_R-dimensional Riemann surface. The first term of (12) is associated with the zero temperature vacuum energy, the second one describes the thermal corrections due to the spatial zero mode (when present, if not so, this term has just to be deleted), and the third one is the thermal contribution due to real excitations of the conformally coupled fields on S^{d-1}. This simple, and easy to interpret, formula is the end result of the generalised zeta function approach used here.

3 Scalar field on the unidimensional Riemann surface

We consider, to start with, the thermodynamics associated with the zero mode. First of all, let us remark that the thermal energy pertaining to the zero mode does not depend on the scaling mass, in fact

$$E_{Z.M.}\left(\beta\right)=-\frac{d}{d\beta}\log\, Z_{Z.M.}\left(\beta\right)=\frac{d}{d\beta}\log\left(\frac{\mu\beta}{2\pi}\right)=\frac{1}{\beta}. \tag{13}$$

On the contrary, the free energy and the entropy do. The free energy is

$$F_{Z.M.}\left(\beta\right)=-\frac{1}{\beta}\log\, Z_{Z.M.}\left(\beta\right)=\frac{1}{\beta}\log\left(\frac{\mu\beta}{2\pi}\right), \tag{14}$$

and the entropy reads

$$S_{Z.M.}\left(\beta\right)=-\beta^2\frac{d}{d\beta}\frac{1}{\beta}\log\, Z_{Z.M.}\left(\beta\right)=-\log\left(\frac{\mu\beta}{2\pi}\right)+1. \tag{15}$$

Since the thermal energy and the free energy are related by

$$\beta F\left(\beta\right)=\int E\left(\beta\right)+C, \tag{16}$$

we see that, in order to have complete compatibility between (13) and (14), we must necessarily choose $C=\log\left(\mu/2\pi\right)$. One can also verify, quite easily, that the fundamental relation between the three quantities above,

$$S=\beta\left(E-F\right), \tag{17}$$

is actually satisfied by (13), (14) and (15). These results are in complete agreement with the ones obtained by Cardy [5], provided that we choose $\mu = 2\pi$ in inverse length units. Their relevance, however, is somehow dubious. We will discuss this delicate point at the end.

For conformally coupled scalar fields on S^{d-1}, Eq. (12) reads

$$\log\, Z\left(\beta\right) = -\sum_{\ell\in N}\left(\ell+1\right)^{d-2}\log\left(1-e^{-\beta(\ell+1)/a}\right). \tag{18}$$

And, also from (12), the zero temperature vacuum energy is

$$E_0 = \frac{1}{2a}\sum_{\ell\in N\}}\left(\ell+1\right)^{d-2}\left(\ell+1\right) = \frac{1}{2a}\zeta\left(1-d\right). \tag{19}$$

Let us consider the unidimensional Riemann surface for which $d_R = 1$ and $D = d = 2$. In this case

$$E_0 = \frac{1}{2a}\zeta\left(-1\right) = -\frac{1}{24a}. \tag{20}$$

On the other hand, Eq. (18) reads

$$\log\, Z\left(\beta\right) = -\sum_{\ell\in N_0}\log\left(1-e^{-\beta\ell/a}\right), \tag{21}$$

where we have shifted the quantum number ℓ by one unit. Since the thermal energy is given by

$$\widetilde{E}\left(\beta\right) = -\frac{d}{d\beta}\log\, Z\left(\beta\right), \tag{22}$$

one can combine (20), (21) and (22) to obtain the total energy, in the form

$$\begin{aligned} E\left(\beta\right) &= E_0 + \widetilde{E}\left(\beta\right) \\ &= -\frac{1}{24a} + \frac{1}{a}\sum_{\ell\in N}\frac{\ell}{e^{\beta\ell/a}-1}. \end{aligned} \tag{23}$$

One can now make use of an appropriate sum formula, say Euler-MacLaurin, Abel-Plana, or Poisson, in one of its several versions, to evaluate the sum on the rhs of (23). As an alternative to Euler-Maclaurin's formula employed in [1], let us consider here the Abel-Plana sum formula

$$\sum_{n\in N} f\left(n\right) = -\frac{1}{2}f\left(0\right) + \int_0^\infty dx\, f\left(x\right) + i\int_0^\infty dx\,\frac{f\left(ix\right)-f\left(-ix\right)}{e^{2\pi x}-1}. \tag{24}$$

A straightforward application of this expression, with $f(x) = x/\left(e^{\beta x/a} - 1\right)$, leads to

$$E(\beta) = -\frac{1}{2\beta} + \frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^2, \quad a/\beta \gg 1, \tag{25}$$

which is the result obtained in [1] and questioned in [4].

Notice that (23) is more appropriate for a low-temperature representation of the energy, and upon the use of the Abel-Plana sum formula it yields the high-temperature result. As is well known, this is a property of this kind of sum formulas (starting with the Jacobi identity), which relate a series expansion valid for say low values of the relevant parameter with a corresponding series expansion for large values of the parameter. In spite of those formulas being identities (both expansions correspond of course to the *same* function), one should be careful *not* to mix terms of one of the expansions with terms of the other one.

It can be verified precisely that it is the third term in the Abel-Plana formula the one that exactly cancels the zero temperature contribution. In fact, the contribution of this term reads

$$i\int_0^\infty dx\, \frac{f(ix) - f(-ix)}{e^{2\pi x} - 1} = \int_0^\infty dx\, \frac{x}{e^{2\pi x} - 1} = \frac{1}{24}. \tag{26}$$

Observe also that the result given by (25) is closed, in the sense that the use of the Abel-Plana sum formula does not allow for exponential corrections, hence it holds *only* in the very high temperature regime.

4 The low-temperature regime

We will now construct an alternative representation for the thermal energy. Contrary to what has been done in the preceding section, through most of the present one we will be dealing with expressions valid in the high-temperature regime, and towards the end we will make use of the Abel-Plana formula in order to obtain the desired low-temperature expansion.

In order to obtain, to begin with, a high-temperature representation of the free energy, we make use of the Poisson sum formula, in the form

$$\sum_{n\in N_0} f(n) = -\frac{f(0)}{2} + \int_0^\infty dx\, f(x) + 2\sum_{n\in N_0}\int_0^\infty dx\, f(x)\,\cos(2\pi n x), \tag{27}$$

where here $f(x) = \log\left(1 - e^{-\beta x/a}\right)$. A straightforward application of (27) to (21) yields

$$\log Z(\beta) = \frac{f(0)}{2} - I(0) - 2\sum_{n\in N_0} I(n), \tag{28}$$

where

$$I\left(n\right):=\int_{0}^{\infty}dx\,f\left(x\right)\,\cos\left(2\pi nx\right),\qquad n\in N. \tag{29}$$

The integrals defined by $I\left(n\right)$ can be readily evaluated with the help of formulas to be found in [12], and the result is

$$\log\,Z\left(\beta\right)=\frac{f\left(0\right)}{2}+\frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^{2}-\frac{\beta}{24a}+\frac{1}{2}\sum_{n\in N_{0}}\frac{1}{n}\coth\left(\frac{2\pi^{2}na}{\beta}\right). \tag{30}$$

Notice that $f\left(0\right)$ is a formally divergent contribution. If we write

$$\log\left(1-e^{-\beta x/a}\right)=-\sum_{p\in N_{0}}\frac{1}{p}e^{-\beta xp/a}, \tag{31}$$

we can see that

$$f\left(0\right)=-\zeta\left(1\right). \tag{32}$$

On the other hand, we can also make use of the expansion

$$\coth\left(x\right)=1+2\sum_{p\in N_{0}}e^{-2px},\qquad x>0. \tag{33}$$

As a consequence, the formally divergent constant cancels out and we obtain the finite result

$$\log\,Z\left(\beta\right)=\frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^{2}-\frac{\beta}{24a}+\frac{1}{2}\sum_{p,\,n\in N_{0}}\frac{1}{n}\,e^{-4\pi^{2}\,n\,p\,a/\beta}. \tag{34}$$

It follows that the thermal part of the free energy in this representation is given by

$$\begin{aligned}\widetilde{F}\left(\beta\right)&=-\frac{1}{\beta}\log\,Z\left(\beta\right)\\&=-\frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^{2}+\frac{1}{24a}-\frac{1}{\beta}\sum_{p,\,n\in N_{0}}\frac{1}{n}\,e^{-4\pi^{2}\,n\,p\,a/\beta}.\end{aligned} \tag{35}$$

This result fulfills pertinent relations given in [2], namely: that the high temperature expansion of the free energy must have the general form

$$-\widetilde{F}\left(\beta\right)a=a_{d}\left(\frac{2\pi a}{\beta}\right)^{d}+a_{d-1}a_{d}\left(\frac{2\pi a}{\beta}\right)^{d-2}+\cdots+a_{0}\left(\frac{2\pi a}{\beta}\right)^{0}+O\left(e^{-4\pi^{2}a/\beta}\right), \tag{36}$$

and that the coefficients of this expansion must satisfy the relation

$$\sum_{k=0}^{d/2}\left(-1\right)^{k}\left(2k-1\right)a_{2k}=0. \tag{37}$$

Note that the inclusion of the zero mode would spoil those relations. The total free energy is

$$F(\beta) = E_0 + \widetilde{F}(\beta) = -\frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^2 - \frac{1}{\beta}\sum_{p,\,n\in N_0}\frac{1}{n}\,e^{-4\pi^2\,n\,p\,a/\beta}. \tag{38}$$

The total energy reads

$$E(\beta) = \frac{d}{d\beta}\left(\beta F(\beta)\right) = \frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^2 - \frac{4\pi^2 a}{\beta^2}\sum_{p,\,n\in N_0} p e^{-\left(\frac{4\pi^2\,n\,p\,a}{\beta}\right)}, \tag{39}$$

and can be summed over n to yield the high temperature representation

$$E(\beta) = \frac{1}{24a}\left(\frac{2\pi a}{\beta}\right)^2 - \frac{4\pi^2 a}{\beta^2}\sum_{p\in N_0}\frac{p}{e^{-4\pi^2 pa/\beta} - 1}. \tag{40}$$

If we now, as before, make use of the Abel-Plana sum formula (24), we obtain

$$E(\beta) = -\frac{1}{24a} + \frac{1}{2\beta}, \qquad a/\beta \ll 1\,. \tag{41}$$

Notice that, as announced above, we have here started from a high-temperature representation for the energy and now, upon the use of the Abel-Plana sum formula, we obtain the energy in the low-temperature limit, a procedure that can be considered the reciprocal of the one employed in Ref. [1]. Notice also that it is the third term in the Abel-Plana formula the one that cancels out the Stefan-Boltzmann term. Therefore, here the term claimed in [4] appears in the low temperature limit *only* and there is *no* cancellation in the high temperature regime if, from the start, we do *not* consider the zero mode.

5 Final remarks

In this brief analysis we have focused on the possible contribution of the spatial zero mode to the thermal energy of a scalar field in a $S^1 \times S^1$ geometry. We have shown that, though the thermal energy does not depend on the scaling mass μ, the partition function, the free energy and the entropy do depend on the choice of value for this parameter. However, the important obstruction to be circumvented is to satisfy the third law of thermodynamics for, as it stands, the entropy (15) associated with the spatial zero mode diverges in the zero temperature limit, a fact also noticed in [1]. It was argued in [8] that the scaling mass can be redefined. Unfortunately, any tentative of redefining the scaling mass, that is, of eliminating the scaling mass dependence from (15) will lead to clear contradictions among the thermodynamical relations linking the basic quantities (13), (14) and (15). For the sake of the argument, suppose for instance that we set this term equal to unity. Then the fundamental relation (17) will be satisfied but the relation $E = d(\beta F)/d\beta$ for the zero mode will be not. Hence though the thermal energy does not depend on the scaling mass, it seems that coherence forces us to set the zero mode and its thermodynamics aside. This neglection of the spatial zero mode should thus be considered as an unavoidable physical requirement and not as a matter of pure convention, that could simply be changed. Notice also that the thermal energy, the entropy and the free energy of the zero mode do not depend on the radius of the Riemann surface. This means that (at least at the one-loop level) their contribution to any isothermal process —e.g., the only ones permitted if the system is in thermal equilibrium with a heath bath— will not be physically observable. This also holds for the linear contribution at high temperature, though in this limit there are no issues concerning the behavior of the equilibrium thermodynamical quantities.

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References

[1] I. Brevik, K.A. Milton and S.D. Odintsov: *Entropy Bounds in $R \times S^3$ Geometries*, hep-th/0202048 v3.

[2] D. Kutasov and F. Larsen, *Partition Sums and Entropy Bounds in Weakly Coupled CFT*, hep-th/0009244.

[3] D. Klemm, A.C. Petkou and G. Siopsis, Nucl. Phys. ***B* 601** (2000) 380.

[4] J.S. Dowker, *Zero modes, entropy bounds and partition functions*, hep-th/0203026.

[5] J.L. Cardy, Nucl. Phys. **B 366** (1991), 403.

[6] E. Verlinde, hep-th/00084140.

[7] J.L. Cardy, Nucl. Phys. **B 270** (1986), 186.

[8] J.S. Dowker: *Class. Quant. Grav.* **1** (1984), 359.

[9] J.S. Dowker and K. Kirsten, *Comm. in Anal. and Geom.* **7** (1999), 641.

[10] E. Elizalde, S.D. Odintsov, A. Romeo, A.A. Bytsenko and S. Zerbini, *Zeta function regularization techniques with applications* (World Scientific, Singapore, 1994); E. Elizalde: *Ten Physical Applications of Spectral Zeta Functions* (Springer-Verlag, Berlin, 1995).

[11] K. Kirsten and E. Elizalde, Phys. Lett. **B 365** (1996), 72.

[12] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. (Academic Press, San Diego, 1994).